Bruno POIZAT. Logicians and Geometers on Algebraic Groups

Algebraic groups are defined in Algebraic Geometry in a very specific way, permitting them to produce a *Group scheme* whose rational points over an arbitrary field K form a group G_K ; the only constraint is that K contains the parameters involved in the definition of the group. The geometric isomorphisms, allowing Geometers to identify two algebraic groups, are similarly defined in a very restricted manner.

On the other hand, the Model Theorists, while working in an algebraically closed field K , consider the groups and the group-homomophisms which are definable in K ; they are called *constructible* by the Geometers. In some sense, constructible groups are the same as algebraic groups, since it is a theorem that a constructible group is constructibly isomorphic to an algebraic group, which is unique in characteristic 0 , when any constructible group-homomorphism between algebraic groups is in fact geometric.

But this is no more the case in characteristic p , where two algebraic groups may be constructibly isomorphic without being geometrically isomorphic: in other words, they have the same rational points over every algebraically closed field, but not over every field. They are considered as the same object by Model Theorists, but not by Geometers. This phenomenon may be a source of confusion when we transfer a result from one domain to the other.

In this talk I will approach the simple algebraic groups from a model theoretic perspective, keeping in mind the *Algebraicity Conjecture* of Zilber and Cherlin, which is open since 50 years, and which claims that they represent the general case of simple groups of finite Morley rank. We shall study the connection between the automorphisms of the simple group G and the automorphisms of the base field K .

The essential fact is that such a group form what I call an *Autonomous constructible structure.* In characteristic 0 , in any such structure S we can define without parameters a copy L of the base field, and as a consequence we show that the group of constructible automorphisms of S is definable, and is the largest superstable group of automorphisms of S .

In characteristic p , we can define without parameters only a *multifield* $(L_1 , ... L_n)$ of copies of K , but there exists in this case also a maximal definable group $\mathrm{Aut}_{max}(S)$ of automorphisms of S , which is also its maximal superstable group of automorphisms.

We obtain a constructible version of Borel-Tits Theorem: if two autonomous contructible structures, and in particular two simple algebraic groups, defined over the same algebraically closed field K , are abstractly isomorphic, they are constructively isomorphic.

All the results above are obtained by general methods from Model Theory, but to identify, in the case of an algebraic simple group G , the group $\mathrm{Aut}_{max}(G)$ with the group of geometric automorphisms of G , we need a deep result from Algebraic

Geometry, namely the Theorem of Borel and Tits, taking into full account the special isogenies discovered by Chevalley.

References

Paramètres dans les corps algébriquement clos, *to appear in Annales Mathématiques du Québec*

Logique et Géométrie, *to appear in Béziau Festschrift, Springer*

Logiciens et Géomètres : deux points de vue sur les groupes algébriques, *to appear in Atlantika*