SIMPLE FINITE NOVIKOV CONFORMAL ALGEBRAS

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Novikov algebras emerged in the works by I. M. Gelfand, I. Ya. Dorfman (1979) devoted to formal variational calculus and by A. A. Balinskii, S. P. Novikov (1985) who developed an analogue of the Hamiltonian formalism for partial differential equations of hydrodynamic type. By definition, a Novikov algebra is a linear space equipped with bilinear multiplication $\circ$ satisfying the following identities:

$$(a \circ b) \circ c - a \circ (b \circ c) = (b \circ a) \circ c - b \circ (a \circ c),$$
$$(a \circ b) \circ c = (a \circ c) \circ b.$$

The structure theory of this class of algebras started developing from a paper by E. Zelmanov (1987) even prior to the appearance of the name "Novikov algebras" (which was proposed by M. Osborn in 1990). Later, the structure and combinatorial theory of Novikov algebras was developed in a number of works, including (but not limited to) the papers by V. T. Filippov, X. Xu, M. Osborn, A. S. Dzhumadil'daev, L. A. Bokut, Y. Chen, I. P. Shestakov, Z. Zhang, V. N. Zhelyabin, A. P. Pozhidaev, etc.

The theory of conformal algebras appeared in quantum physics as a formalization of the properties of the operator product expansion in the 2-dimensional conformal field theory. From the algebraic point of view, a conformal algebra is a collection of formal distributions (two-sides infinite formal power series) satisfying certain conditions (V. G. Kac, 1996).

If a conformal algebra is represented by formal distributions over a Lie algebra then it is said to be a Lie conformal algebra. A conformal algebra is said to be finite if it is spanned by all formal derivatives of a finite number of distributions.

For example, let $W = \operatorname{Der} \Bbbk[t, t^{-1}]$ be the Lie algebra of all derivations of the (commutative) algebra of Laurent polynomials. Then the only formal distribution

$$v(z) = \sum_{n\in\mathbb{Z}} t^n \partial_t z^{-n-1} \in W[[z, z^{-1}]]$$

along with its formal derivatives with respect to z span a Lie conformal algebra known as the Virasoro conformal algebra Vir.

The structure theory of finite Lie conformal algebras was developed by A. D'Andrea and V. G. Kac (1998). It turns out that Vir is the only exceptional simple finite Lie conformal algebra (over an algebraically closed field of zero characteristic) apart from the family of current conformal algebras $\operatorname{Cur}\mathfrak{g}$ generated by the series

$$a(z) = \sum_{n\in\mathbb{Z}} at^n z^{-n-1} \in \mathfrak{g}[t,t^{-1}][[z,z^{-1}]], \quad a \in \mathfrak{g},$$

where $\mathfrak{g}$ is a finite-dimensional simple Lie algebra.

In this work, we describe simple finite Novikov conformal algebras. For example, let $V = \Bbbk[t,t^{-1}]$ be the algebra of Laurent polynomials with the ordinary derivation ∂_t. Then for every $\alpha \in \Bbbk$ the new operation

$$f \circ g = f\partial_t(g) + \alpha fg, \quad f, g \in V,$$

turns V into a Novikov algebra V_α. The formal distribution

$$v(z) = \sum_{n\in\mathbb{Z}} t^n z^{-n-1} \in V_\alpha[[z,z^{-1}]]$$

generates a Novikov conformal algebra denoted $\mathfrak{V}_\alpha$.

Theorem (joint with J. Liu)**.** *A simple finite Novikov conformal algebra over an algebraically closed field* $\Bbbk$ *of zero characteristic is isomorphic either to* $\operatorname{Cur}\Bbbk$ *or to* $\mathfrak{V}_\alpha$ *for some* $\alpha \in \Bbbk$.

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