LOGICAL TOPOLOGIES AND SEPARATION AXIOMS IN THE INSTITUTION-INDEPENDENT MODEL THEORY FRAMEWORK

Asterios Gkantzounis *

Department of Mathematics
School of Applied Mathematical and Physical Sciences
National Technical University of Athens
Athens, Greece
e-mail: minasteris@gmail.com

Foivos Skarpelos †

Department of Mathematics
School of Applied Mathematical and Physical Sciences
National Technical University of Athens
Athens, Greece
e-mail: f-skarp@hotmail.com

In this paper we study the behavior of various topologies on classes of models and their mappings under model functors in the abstract model-theoretic framework of Institution theory.

The theory of Institutions (otherwise known as "Institution-Independent Model Theory") constitutes the main branch of categorical abstract model theory, and formalizes the notion of a logical system, including syntax, semantics, and the satisfaction relation between them, without being restricted by the concrete characteristics of any particular logical system (for example, First Order Logic and its extensions) [1]. The main advantages of this approach is the aforementioned base logic-independence, the ability to preserve results under change of notation, and the ability to study different logical systems in a uniform manner.

In this abstract, we propose the study of logic and topology independently of the underlying logic, through Institutions. For every signature Σ we study three different classes of topologies over the category of models $\mathbf{Mod}(\Sigma)$. Two of these classes are based on Lewitzka's approach in Universal logic [7, 8], where he defines for every logical system a "natural" topology within the system's theory set, i.e. the *Semantic Topology (SM)*. We study *SM*, whose base in the institutional context is defined over the finite subsets

*Supported by PEVE Universal Logic.
†Supported by PEVE Universal Logic.

of $\boldsymbol{Sen}(\Sigma)$[1], and the *Identification Semantic Topology (ISM)* [6], which is proven to be the Kolmogorov Quotient of *SM*. The third class of topologies is generic (i.e. defined over an arbitrary base, with no further restrictions), and is used as a general object for comparison and reference. We derive results on their behavior and their relation to the separation axioms, expanding on past results [5, 6].

Furthermore, we consider the functors between model categories $\mathbf{Mod}(\phi) : \mathbf{Mod}(\Sigma') \to \mathbf{Mod}(\Sigma)$ induced by translations, which are signature morphisms $\phi : \Sigma \to \Sigma'$. Finally, we investigate the necessary conditions needed for topologies (and the model functors), such that topological structure and specific properties of these topologies are preserved under $\mathbf{Mod}^{-1}(\phi)$ and $\mathbf{Mod}(\phi)$.

Our findings generalize previous results from [3, 6, 10], and build upon and advance our earlier work presented in [4]. The novel framework we establish in our line of study [2, 9], which examines the properties of logical topologies in the institution-theoretic setting, contributes to the ongoing development at the interface of mathematical logic and topology, by highlighting the affinity between topology and formal semantics.

References

[1] Diaconescu, R.: Institution-independent Model Theory (2nd ed.). Birkhauser (2025)

[2] Gkantzounis, A.: Algebraic and geometric methods for the modeling of systems. PhD in progress, Athens, NTUA, Department of Mathematics.

[3] Gkantzounis, A., Stefaneas, P.: Topological Inquiry in Abstract Model Theory. In: Ţuţu, I. (eds) Recent Trends in Algebraic Development Techniques. WADT 2024. Lecture Notes in Computer Science, vol 15587. Springer (2025)

[4] Gkantzounis, A., Skarpelos, F., Kiouvrekis, Y., Stefaneas, P.: On abstract model theory and logical topologies. TACL 2024: Topology, Algebra, and Categories in Logic, Barcelona, Spain (2024)

[5] Kiouvrekis, Y.: Topological models, computer science semantics and applications of logic in ai. PhD Thesis, Athens, NTUA, Department of Mathematics, (2019)

[6] Kiouvrekis, Y., Stefaneas, P.: Topological semantics in institutions with proofs. In: Pinus. A.G., Ponomaryov, K.N., Sudoplatov, S.V., Timo-

shenko, E.I. (eds) Algebra and Model Theory 10, pp. 92–100, NSTU, Novosibirsk (2015)

[7] Lewitzka, S.: Abstract logics, logic maps, and logic homomorphisms. Log. Univers., 1(2):243–276 (2007)

[8] Lewitzka, S: A topological approach to universal logic: Model-theoretical abstract logics. In: Beziau, J.-Y. (eds) Logica Universalis. Birkhäuser (2007)

[9] Skarpelos, F.: Abstract Model Theory and Applications to Computing. PhD in progress, Athens, NTUA, Department of Mathematics.

[10] Sudoplatov, S.V., Kiouvrekis, Y., Stefaneas, P.: Generic constructions and generic limits. Algebraic Modeling of Topological and Computational Structures and Applications, pp. 375–398 (2017)