The isomorphism problem for groups acting on trees

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By the Bass-Serre theorem [1], every group acting on a tree can be represented as $\pi_1(\mathbb{A})$, the fundamental group of a graph of groups $\mathbb{A}$. The isomorphism problem for such groups can be posed as follows: to determine algorithmically when two given graph of groups define isomorphic groups. In general, this is a rather hard problem. We will give an overview of this problem and discuss some new results.

Call a finitely generated group G a *generalized Baumslag-Solitar group* or a GBS group if G can act on a tree so that the stabilizers of vertices and edges are infinite cyclic groups. Given a GBS group G, we can present the corresponding graph of groups $\mathbb{A}$ by a labeled graph (A, λ), where A is a finite connected graph and $\lambda\colon E(A) \to \mathbb{Z} \setminus \{0\}$ labels the edges of A. Every GBS group can be presented by infinitely many labeled graphs.

Recently GBS groups have been actively studied [2], [3], [4]. In particular, the isomorphism problem for GBS groups has been discussed. Despite that, the isomorphism problem is solved only in several special cases [5], [6], [7], [8] and the general solution is not established.

A finitely generated group G_n that acts on a tree such that all edge stabilizers are infinite cyclic groups and all vertex stabilizers are free Abelian groups of rank n will be called a *generalized Baumslag–Solitar group of type (n,1)* ($GBS(n,1)$ group). We will discuss the relationship between the isomorphism of $GBS(n,1)$ groups and GBS groups.

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