HOCHSCHILD COHOMOLOGY OF THE WEYL CONFORMAL ALGEBRA

HASSAN ALHUSSEIN[1], PAVEL KOLESNIKOV[2]

For every conformal Lie algebra L one can construct a series of universal enveloping associative conformal algebras corresponding to different associative locality functions on the generators [5]. For example, consider the Virasoro conformal algebra Vir which is generated by a single element v. One may fix a natural number N and construct the associative conformal algebra $U(N)$ generated by the element v such that $(v_{(n)} v) = 0$ for $n \geq N$, and the commutation relations of Vir hold. Obviously, $U(1) = 0$; the algebra $U(2)$ is known as the Weyl conformal algebra (also denoted $\mathrm{Cend}_{1,x}$ in [2]).

It was shown in [4] that the second Hochschild cohomology groups $\mathrm{H}^2(U(2), M)$ are trivial for every conformal (bi-)module M, but for higher Hochschild cohomologies the direct computation becomes too complicated. In contrast to the "ordinary" Hochschild cohomology, if C is an infinite associative conformal algebra one cannot reduce the computation of $\mathrm{H}^n(C, M)$ to $\mathrm{H}^{n-1}(C, \mathrm{Chom}(C, M))$ since the space of conformal homomorphisms $\mathrm{Chom}(C, M)$ is not in general a conformal module over C.

In this paper we find all higher Hochschild cohomology groups $\mathrm{H}^n(U(2), M)$, $n \geq 2$, of the Weyl conformal algebra $U(2)$ with coefficients in all finite modules M. In order to obtain this result we construct the Anick resolution for its coefficient algebra via the algebraic discrete Morse theory as presented, for example, in [3]. It is discussed in [1] how to adjust this technique for differential algebras to calculate Hochschild cohomologies with coefficients in a trivial module. The purpose of our work is to apply the Morse matching method for calculation of Hochschild cohomologies of associative conformal algebras with coefficients in a non-trivial module. As a result, we find that all Hochschild cohomology groups of $U(2)$ with coefficients in a finite module are trivial except the first one.

References

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[1)] Novosibirsk State Technical University, Novosibirsk, Russia.
Email address: hassanalhussein2014@gmail.com

[2)] Sobolev Institute of Mathematics, Novosibirsk, Russia.
Email address: pavelsk77@gmail.com