Companions of hybrids of fragments of definable subsets

M.T. Kassymetova, N.M. Mussina, N.P. Popova

Karaganda Buketov University
Karagandy, Kazakhstan

14TH INTERNATIONAL SUMMER SCHOOL-CONFERENCE ON THE OCCASION OF THE 75TH BIRTHDAY OF PROFESSOR BRUNO POIZAT

ERLAGOL, June 23-29, 2021

(1) Yeshkeyev A.R., Kasymetova M.T. *Jonsson theories and their classes of models, Karaganda: Izdatelstvo KarGU, 2016. - P. 370. [in Russian]*

(2) *Yeshkeyev A.R., Mussina N.M. Properties of hybrids of Jonsson theories, Bulletin of the Karaganda University. - Seria of "Mathematics". - Karaganda, 2018. - №4 (92). - P. 99-104.*

Definition 1. [1]

A theory T is Jonsson if:
1) Theory T has infinite models;
2) Theory T is inductive;
3) Theory T admits the joint embedding property (JEP);
4) Theory T admits the property of amalgam (AP).

Examples of Jonsson theories are:
1) Group Theory,
2) Theory of Abelian groups,
3) Theory of fields of fixed characteristics,
4) Theory of Boolean algebras,
5) Theory of polygons over a fixed monoid,
6) Theory of modules over a fixed ring,
7) Theory of linear order.

Definition 2. [1]

Model C of Jonsson theory T is called semantic model, if it is ω^+-homogeneous-universal.

Definition 3. [1]

Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^*.

Definition 4. [1]

The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^*, i.e. $T^* = Th(C)$.

Theorem 1 [A.R. Yeshkeyev] [1].

Let T be a Jonsson theory. Then the following conditions are equivalent:
1) Theory T is perfect;
2) Theory T^* is a model companion of theory T.

Theorem 2 [A.R. Yeshkeyev] [1].

If T is a perfect Jonsson theory then $E_T = ModT^*$.

Definition 5. [A.R. Yeshkeyev] [1]

Let $X \subseteq C$. We will say that a set X is $\nabla - cl$-Jonsson subset of C, if X satisfies the following conditions:
1) X is ∇-definable set (this means that there is a formula from ∇, the solution of which in the C is the set X, where $\nabla \subseteq L$, that is ∇ is a view of formula, for example $\exists, \forall, \forall\exists$ and so on.);
2) $cl(X) = M$, $M \in E_T$, where cl is some closure operator defining a pregeometry over C (for example $cl = acl$ or $cl = dcl$).

Lemma 1 [1].

Let T be Jonsson theory, E_T be the class of its existentially closed models. Then for any model $A \in E_T$ the theory $Th_{\forall\exists}(A)$ is a Jonsson theory.

Let X_1, X_2 be ∇-cl-Jonsson subsets of C, where C is the semantic model of the theory T. Let $M_1 = cl(X_1)$, $M_2 = cl(X_2)$, where $M_1, M_2 \in E_T$. $Th_{\forall\exists}(M_1) = Fr(X_1)$, $Th_{\forall\exists}(M_2) = Fr(X_2)$. C_1 is semantic model of theory $Fr(X_1)$, C_2 is semantic model of theory $Fr(X_2)$.

We define the essence of the operation of the symbol $\diamond$ for algebraic construction of models, which will be play important role in the definition of hybrids. Let $\diamond \in \{\times, +, \oplus, \prod\limits_F, \prod\limits_U\}$, where $\times$ is Cartesian product, $+$ is the sum and $\oplus$ is the direct sum, $\prod\limits_F$ is reduced product and $\prod\limits_U$ is the ultraproduct of models.

Definition 6.[A.R. Yeshkeyev][2]

Let $Fr(X_1), Fr(X_2)$ be the fragments. A hybrid $H(Fr(X_1), Fr(X_2))$ of fragments $Fr(X_1)$, $Fr(X_2)$ is called the theory $Th_{\forall\exists}(C_1 \times C_2)$, if it is Jonsson theory, where C_i are the semantic models of $Fr(X_i), i = 1, 2$.

Fact [2].

For the theory $H(Fr(X_1), Fr(X_2))$ to be a Jonsson theory enough to $(C_1 \times C_2) \in E_T$.

The following examples will be examples of hybrids of Jonsson theories:

- Let T be a Jonsson theory, C is the semantic model of the theory T. A, B are the Jonsson subsets, $A, B \subseteq C$. $dcl(A) = M_1$, $dcl(B) = M_2$, where $M_1, M_2 \in E_T$. Then $Th_{\forall\exists}(M_1 \times M_2)$ will be a hybrid of Jonsson theories.
- Let T_1, T_2 be the Jonsson theories of Abelian groups, C_1, C_2 are the semantic models of the theories T_1, T_2, respectively. Then $Th_{\forall\exists}(C_1 \times C_2) = H(T_1, T_2)$ will be a hybrid of Jonsson theories.
- Let V be a linear space, V_1, V_2 be the linear subspaces, $V_1, V_2 \subseteq V$. Then $Th_{\forall\exists}(V_1 \oplus V_2)$ will be a hybrid of Jonsson theories.

Definition 7.[A.R. Yeshkeyev]

The inductive theory T is called the existentially prime if: it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T; class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq \emptyset$.

Model consistent theory

Definition 8. [1]

T and T^* are mutually model consistent, i.e. every model of theory T is embeddable in a model of theory T^*, and vice versa.

Let T be a Jonsson theory. C is semantical model of the theory T. $X \subseteq C$.

Theorem 1.

Let $Fr(X)$ be a pecfect convex existentially prime complete for $\forall\exists$-sentences a Jonsson theory. X_1, X_2 are the Jonsson sets of the theory $Th_{\forall\exists}(C)$, where $M_i = dcl(X_i) \in E_{Fr(Th_{\forall\exists}(C))}, Fr(X_i) = Th_{\forall\exists}(M_i)$ are also perfect convex existentially prime complete for $\forall\exists$-sentences a fragments. Then, if their hybrid $H(Fr(X_1), Fr(X_2))$ is a model consistent with $Fr(X_i)$, then $H(Fr(X_1), Fr(X_2))$ is a perfect Jonsson theories for $i = 1, 2$.

Theorem 2.

Let $Fr(X), Fr(X_1), Fr(X_2)$ satisfy the conditions of Theorem 1 and $Fr(X_1)$, $Fr(X_2)$ be ω-categorical fragments. Then their hybrid $H(Fr(X_1), Fr(X_2))$ is also a ω-categorical Jonsson theory.

Thank you for your attention!