MODEL-THEORETICAL AND TOPOLOGICAL PROPERTIES OF FAMILIES OF THEORIES

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Goals and objectives

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Families of complete theories. Bibliography

- S. V. Sudoplatov, "Combinations of structures", Известия Иркутского государственного университета. Серия Математика, 24 (2018), 82–101
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- S. V. Sudoplatov, "Relative e-spectra and relative closures for families of theories", Сиб. электрон. матем. изв., 14 (2017), 296–307
- S. V. Sudoplatov, "On semilattices and lattices for families of theories", Сиб. электрон. матем. изв., 14 (2017), 980–985
- S. V. Sudoplatov, "Approximations of theories", Сиб. электрон. матем. изв., 17 (2020), 715–725
- S. V. Sudoplatov, Ranks for families of theories and their spectra, Lobachevskii Journal of Mathematics (to appear). arXiv:1901.08464v1[math.LO]. 2019
- Ю. Л. Ершов, Топология для дискретной математики / Ю.Л. Ершов; отв. ред. В.Л. Селиванов; Рос. акад. наук, Сиб. отд-ние, Ин-т математики им. С.Л. Соболева. – Новосибирск: Изд-во СО РАН, 2020. – 333 с.

φ-neighborhoods of families of complete theories

We consider the families $\mathcal{T}$ of complete first-order theories of the language $\Sigma = \Sigma(\mathcal{T})$.

- **Definition.**[1] We denote by $\mathcal{T}_\varphi$ the set $\{T \in \mathcal{T} \mid \varphi \in T\}$.
- Any set $\mathcal{T}_\varphi$ is called *φ-neighborhood* or just *neighborhood* for $\mathcal{T}$ or a *(φ-)definable* subset of $\mathcal{T}$.
- The set $\mathcal{T}_\varphi$ is also called *definable (by a formula or sentence φ)* with respect to $\mathcal{T}$, or *s-definable*.

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- **Definition.**[2] For the empty family $\mathcal{T}$ we put the rank $\mathrm{RS}(\mathcal{T}) = -1$, for finite nonempty families $\mathcal{T}$ we put $\mathrm{RS}(\mathcal{T}) = 0$, and $\mathrm{RS}(\mathcal{T}) \geq 1$ for infinite families $\mathcal{T}$.
- For a family $\mathcal{T}$ and an ordinal $\alpha = \beta + 1$ we put $\mathrm{RS}(\mathcal{T}) \geq \alpha$ if there are pairwise inconsistent $\Sigma(\mathcal{T})$-sentences φ_n, $n \in \omega$, such that $\mathrm{RS}(\mathcal{T}_{\varphi_n}) \geq \beta$, $n \in \omega$.
- If α is a limit ordinal, then $\mathrm{RS}(\mathcal{T}) \geq \alpha$ if $\mathrm{RS}(\mathcal{T}) \geq \beta$ for any $\beta < \alpha$.
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Degree for families of complete theories

- **Definition.**[3] A family $\mathcal{T}$ is called *e-totally transcendental*, or *totally transcendental*, if $\mathrm{RS}(\mathcal{T})$ is an ordinal.
- If $\mathcal{T}$ is totally transcendental, with $\mathrm{RS}(\mathcal{T}) = \alpha \geq 0$, we define the *degree* $\mathrm{ds}(\mathcal{T})$ of $\mathcal{T}$ as the maximal number of pairwise inconsistent sentences φ_i such that $\mathrm{RS}(\mathcal{T}_{\varphi_i}) = \alpha$.

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Approximations for families of complete theories

- **Definition.**[4] Let $\mathcal{T}$ be a family of theories and T be a theory, $T \notin \mathcal{T}$. The theory T is called *$\mathcal{T}$-approximated*, or *approximated by $\mathcal{T}$*, or *$\mathcal{T}$-approximable*, or a *pseudo-$\mathcal{T}$-theory*, if for any formula $\varphi \in T$ there is $T' \in \mathcal{T}$ such that $\varphi \in T'$.
- If T is $\mathcal{T}$-approximated then $\mathcal{T}$ is called an *approximating family* for T, theories $T' \in \mathcal{T}$ are *approximations* for T, and T is an *accumulation point* for $\mathcal{T}$.
- An approximating family $\mathcal{T}$ is called *e-minimal* if for any sentence $\varphi \in \Sigma(\mathcal{T})$, $\mathcal{T}_\varphi$ is finite or $\mathcal{T}_{\neg\varphi}$ is finite.

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Closure for families of complete theories

- Recall that the E-closure $Cl_E(\mathcal{T})$[5] for the family $\mathcal{T}$ of complete theories is characterized by the following proposition.
- **Proposition.** *Let $\mathcal{T}$ be a family of complete theories of the language Σ. Then $Cl_E(\mathcal{T}) = \mathcal{T}$ for finite $\mathcal{T}$ and for infinite $\mathcal{T}$, the theory T belongs to $Cl_E(\mathcal{T})$ if and only if T is a complete theory of the language Σ and $T \in \mathcal{T}$, or $T \notin \mathcal{T}$ and for of any formula $\varphi \in T$ the set $\mathcal{T}_\varphi$ is infinite.*

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Regularly closed, pseudo-algebraically closed, quasifinite fields

It should be noted are actively developing in recent area of study **pseudofinite** structures.

- **F is regularly closed,**[6] *if every absolutely irreducible variety V over F has a point defined over F.*
- Ax fields, Σ-fields, **Pseudo-algebraically closed fields**[7]
- **F is quasifinite**, if it is perfect and has exactly one extension of each degree (in the fixed algebraic closure $\tilde{F}$).

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Regularly closed fields. Bibliography

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Pseudo-algebraically closed fields. Bibliography

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Pseudofinite theories

- **Theorem.**(Ax,1968)The field F is *pseudofinite* if and only if all of the following conditions are met:
  (i) F is perfect;
  (ii) F is quasifinite;
  (iii) F is pseudo-algebraically closed (PAC)[8] [9].
- **Proposition.** (Ax,1968) *We fix the language Σ and the Σ-structure $\mathcal{M}$. Then the following are equivalent:*
  1. *The Σ-structure $\mathcal{M}$ is pseudofinite;*
  2. *$\mathcal{M} \models T_f$, where T_f is the general theory of all finite Σ-structures;*
  3. *$\mathcal{M}$ is elementarily equivalent to an ultraproduct of finite Σ-structures.*
- **Definition.**[10] *A Σ-structure $\mathcal{M}$ is called **pseudofinite** if, for all Σ-sentences φ, it follows from $\mathcal{M} \models \varphi$ that there is a finite $\mathcal{M}_0$ such that $\mathcal{M}_0 \models \varphi$.*

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Some examples

- DLO theory is not pseudofinite
- The Abelian group $(\mathbb{Z}, +)$ is not pseudofinite.
- The theory of algebraically closed fields is not pseudofinite

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Pseudofinite structures (fields, groups, rings). Bibliography

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Basic results on pseudofinite theories

- **Theorem.** [11] The theory of any pseudo-algebraically closed (PAC) field that is not separably closed has the independence property. (That is, the pseudo-finite theory is unstable)
- **Theorem.** [12] Totally categorical theories (and more generally ω-categorical ω-stable theories[13]) are pseudofinite.

[11] Duret, J. Les corps pseudo-finis ont la propriete d'independance. C. R. Acad. Sei. Paris Ser. 1980. Vol. 290. P. 981—903.

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Basic results on pseudofinite theories

- **Theorem.** [11] The theory of any pseudo-algebraically closed (PAC) field that is not separably closed has the independence property. (That is, the pseudo-finite theory is unstable)
- **Theorem.** [12] Totally categorical theories (and more generally ω-categorical ω-stable theories[13]) are pseudofinite.

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Ranks for families of theories of arbitrary languages(with S.V.Sudoplatov)[14]

Let Σ be a language. If Σ is relational, then by $\mathcal{T}_\Sigma$ we denote the family of all theories of the language Σ.

- **Theorem 1.** For any language Σ the family $\mathcal{T}_\Sigma$ is *e*-minimal if and only if $\Sigma = \emptyset$ or Σ consists of one constant symbol.
- **Theorem 2.** For any language Σ either $\mathrm{RS}(\mathcal{T}_\Sigma)$ is finite, if Σ consists of finitely many 0-ary and unary predicates, and finitely many constant symbols, or $\mathrm{RS}(\mathcal{T}_\Sigma) = \infty$, otherwise.

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Calculi for definable families of complete theories (with S.V.Sudoplatov)[15]

- **Definition.** For a family $\mathcal{T}$ and sentences φ and ψ we say that φ $\mathcal{T}$*-forces* ψ, written $\varphi \vdash_{\mathcal{T}} \psi$ if $\mathcal{T}_\varphi \subseteq \mathcal{T}_\psi$.
- Definition. If $\mathcal{T}$ is a family of theories and Φ is a set of sentences, then we put $\mathcal{T}_\Phi = \bigcap\limits_{\varphi\in\Phi} \mathcal{T}_\varphi$ and the set $\mathcal{T}_\Phi$ is called (*type-* or *diagram-*)*definable* (by the set Φ) with respect to $\mathcal{T}$, or (*diagram-*)$\mathcal{T}$*-definable*, or simply *d-definable*.
- If Φ is a singleton $\{\varphi\}$, then $\mathcal{T}_\varphi = \mathcal{T}_\Phi$ is called *s-definable*.
- **Proposition 3.** *For any sets Φ and Ψ of sentences and a family $\mathcal{T}$ of theories the following conditions are equivalent:*
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Compactness of definable families of complete theories(with S.V.Sudoplatov)[16]

- **Definition.** A d-definable set $\mathcal{T}_\Phi$ is called *$\mathcal{T}$-consistent* if $\mathcal{T}_\Phi \cap \mathcal{T} \neq \emptyset$, and $\mathcal{T}_\Phi$ is called *locally $\mathcal{T}$-consistent* if for any finite $\Phi_0 \subseteq \Phi$, $\mathcal{T}_{\Phi_0}$ is $\mathcal{T}$-consistent.
- **Theorem 4.** (Compactness theorem) *For any nonempty E-closed family $\mathcal{T}$, every locally $\mathcal{T}$-consistent d-definable set $\mathcal{T}_\Phi$ is $\mathcal{T}$-consistent.*
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- **Theorem 5.** *For any E-closed family $\mathcal{T}$, there is a d-definable family $\mathcal{T}_\Phi$ which is not s-definable if and only if $\mathcal{T}$ is infinite.*

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Algebras for definable subfamilies of complete theories(with S.V.Sudoplatov)[17]

- **Theorem 6.** For any nonempty E-closed family $\mathcal{T}$ and $n \in \omega \setminus \{0\}$ the following conditions are equivalent:
  (1) $\mathrm{RS}(\mathcal{T}) = 1$ and $\mathrm{ds}(\mathcal{T}) = n$;
- (2) *Boolean algebra $\mathcal{B}_s(\mathcal{T})$ is isomorphic to the direct product n of Boolean algebras $\mathcal{B}_1, \ldots, \mathcal{B}_n$, each of which is generated by elements of rank $< \alpha$ such that each $\mathcal{B}_n$ contains infinitely many elements of each rank $\beta < \alpha$;*
- (3) *the algebra $\mathcal{B}_d(\mathcal{T})$ consists of an infinite number of atomic elements of each rank $\beta < \alpha, \beta \geq 0$ and exactly n atomic elements of rank α, these n atomic elements correspond to nonprinciple ultrafilters with respect to elements of rank $< \alpha$.*

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Topologies for families of complete theories(with S.V.Sudoplatov)[18]

- We consider families $\mathcal{T}$ of consistent first-order theories in languages $\Sigma \subseteq \Sigma(\mathcal{T})$, not necessary complete. By $F(\Sigma)$ we denote the set of all formulas in the language Σ and by $\mathrm{Sent}(\Sigma)$ the set of all sentences in $F(\Sigma)$.
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- **Proposition 9.** *A T_1-space $\mathcal{T}$ is Hausdorff if and only if for any distinct theories $T, T' \in \mathcal{T}$ there are sentences $\varphi \in T$, $\psi \in T'$ such that $\varphi \wedge \psi$ is $\mathcal{T}$-inconsistent.*
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$\overline{e}$-minimal and Bs-minimal subfamilies(with S.V.Sudoplatov)[19]

- **Definition.** An infinite family $\mathcal{T}$ of (possibly incomplete) theories is called $\overline{e}$-*minimal* if for any sentence $\varphi \in \mathrm{Sent}(\Sigma(\mathcal{T}))$, $\mathcal{T}_\varphi$ is finite or $\overline{\mathcal{T}_\varphi} = \mathcal{T} \setminus \mathcal{T}_\varphi$ is finite.
- Any boolean combination of *s*-definable subfamilies of $\mathcal{T}$ is reduced to unions of sets of form $\mathcal{T}_\varphi \cap \overline{\mathcal{T}_{\psi_1}} \cap \ldots \cap \overline{\mathcal{T}_{\psi_n}}$, which, in general case, can not be written shorter. These boolean combinations are called Bs-*definable* subfamilies of $\mathcal{T}$.
- For a family $\mathcal{T}$ we denote by $\mathrm{Bs}_{\mathcal{T}}$ the set of all Bs-definable subfamilies of $\mathcal{T}$. Clearly, $\mathrm{Bs}_{\mathcal{T}}$ forms a boolean algebra. Moreover, we have:
- **Lemma 11.** *For any family $\mathcal{T}$ the pair $(\mathcal{T}, \mathrm{Bs}_{\mathcal{T}})$ is Hausdorff.*

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- **Definition.** An infinite family $\mathcal{T}$ of (possibly incomplete) theories is called $\overline{e}$*-minimal* if for any sentence $\varphi \in \mathrm{Sent}(\Sigma(\mathcal{T}))$, $\mathcal{T}_\varphi$ is finite or $\overline{\mathcal{T}_\varphi} = \mathcal{T} \setminus \mathcal{T}_\varphi$ is finite.
- Any boolean combination of *s*-definable subfamilies of $\mathcal{T}$ is reduced to unions of sets of form $\mathcal{T}_\varphi \cap \overline{\mathcal{T}_{\psi_1}} \cap \ldots \cap \overline{\mathcal{T}_{\psi_n}}$, which, in general case, can not be written shorter. These boolean combinations are called Bs-*definable* subfamilies of $\mathcal{T}$.
- For a family $\mathcal{T}$ we denote by $\mathrm{Bs}_\mathcal{T}$ the set of all Bs-definable subfamilies of $\mathcal{T}$. Clearly, $\mathrm{Bs}_\mathcal{T}$ forms a boolean algebra. Moreover, we have:
- **Lemma 11.** *For any family $\mathcal{T}$ the pair $(\mathcal{T}, \mathrm{Bs}_\mathcal{T})$ is Hausdorff.*

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Ranks for families of incomplete theories(with S.V.Sudoplatov)[20]

- **Definition.** For the empty family $\mathcal{T}$ we put the rank $\overline{\mathrm{RS}}(\mathcal{T}) = -1$, and for finite nonempty families $\mathcal{T}$ we put $\overline{\mathrm{RS}}(\mathcal{T}) = 0$.
- For a family $\mathcal{T}$ and an ordinal $\alpha = \beta + 1$ we put $\overline{\mathrm{RS}}(\mathcal{T}) \geq \alpha$ if there are pairwise disjoint Bs-definable subfamilies $\mathcal{T}_n$ of $\mathcal{T}$, $n \in \omega$, such that $\overline{\mathrm{RS}}(\mathcal{T}_n) \geq \beta$, $n \in \omega$.
- If α is a limit ordinal then $\overline{\mathrm{RS}}(\mathcal{T}) \geq \alpha$ if $\overline{\mathrm{RS}}(\mathcal{T}) \geq \beta$ for any $\beta < \alpha$.
- We set $\overline{\mathrm{RS}}(\mathcal{T}) = \alpha$ if $\overline{\mathrm{RS}}(\mathcal{T}) \geq \alpha$ and $\overline{\mathrm{RS}}(\mathcal{T}) \not\geq \alpha + 1$.
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Ranks and degrees for families of incomplete theories(with S.V.Sudoplatov)[21]

- **Definition.** A family $\mathcal{T}$ is called $\overline{e}$*-totally transcendental* if $\overline{\mathrm{RS}}(\mathcal{T})$ is an ordinal.
- If $\mathcal{T}$ is $\overline{\mathrm{e}}$-totally transcendental, with $\overline{\mathrm{RS}}(\mathcal{T}) = \alpha \geq 0$, we define the *degree* $\overline{\mathrm{ds}}(\mathcal{T})$ of $\mathcal{T}$ as the maximal number of pairwise disjoint Bs-definable subfamilies $\mathcal{T}_i$ such that $\overline{\mathrm{RS}}(\mathcal{T}_i) = \alpha$.
- By the definition, if $\overline{\mathrm{RS}}(\mathcal{T}) = \alpha$, for an ordinal α, then $\overline{\mathrm{ds}}(\mathcal{T}) \in \omega \setminus \{0\}$.
- **Theorem 12.** *For any ordinal α and natural $n \neq 0$ there is a family $\mathcal{T}$ of positive theories such that $(\overline{\mathrm{RS}}(\mathcal{T}), \overline{\mathrm{ds}}(\mathcal{T})) = (\alpha, n)$.*

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Algebras for *Bs*-definable subfamilies of families of theories(with S.V.Sudoplatov)[22]

- An intersections, possibly infinite, of Bs-definable subfamilies of $\mathcal{T}$ are called Bs-*diagram definable*, or Bd-*definable*.
- We extend the algebra $\mathcal{B}_{\mathrm{Bs}}(\mathcal{T})$ till the algebra $\mathcal{B}_{\mathrm{Bd}}(\mathcal{T})$ of all Bd-definable subfamilies of $\mathcal{T}$. Clearly, the algebra $\mathcal{B}_{\mathrm{Bd}}(\mathcal{T})$ preserves the operations $\cap$ and $\cup$, whereas complements are defined only for Bs-definable sets. Therefore $\mathcal{B}_{\mathrm{Bd}}(\mathcal{T})$ contains the operations $\cap$ and $\cup$, being a partial algebra with respect to $\bar{\ }$.
- Thus, $\mathcal{B}_{\mathrm{Bd}}(\mathcal{T})$ forms a distributive lattice, partially, for Bs-definable sets, with complements.

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- **Theorem 13.** *For any nonempty* Bs*-closed special family* $\mathcal{T}$*, an ordinal* $\alpha \geq 1$*, and* $n \in \omega \setminus \{0\}$*, the following conditions are equivalent:* (1) $\overline{\mathrm{RS}}(\mathcal{T}) = \alpha$ and $\overline{\mathrm{ds}}(\mathcal{T}) = n$;
- (2) *the Boolean algebra* $\mathcal{B}_{\mathrm{Bs}}(\mathcal{T})$ *is isomorphic to a direct product of* n *Boolean algebras* $\mathcal{B}_1, \ldots, \mathcal{B}_n$ *each of which is generated by elements of ranks* $< \alpha$ *such that each* $\mathcal{B}_n$ *contains infinitely many elements of each rank* $\beta < \alpha$*;*
- (3) *the algebra* $\mathcal{B}_{\mathrm{Bd}}(\mathcal{T})$ *consists of infinitely many atomic elements of each rank* $\beta < \alpha$*, for* $\beta \geq 0$*, and exactly* n *atomic elements of rank* α*, these* n *atomic elements correspond non-principal ultrafilters with respect to elements of ranks* $< \alpha$*.*

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Closures(with S.V.Sudoplatov)[24]

- **Definition.** For a family $\mathcal{T}$ of theories in a language Σ and a theory T we put $T \in \mathrm{Cl}_1(\mathcal{T})$ if $T \in \mathcal{T}$, or T is nonempty and
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- If $\mathcal{T}'$ is fixed then we say that T *belongs to the* Cl_1*-closure of* $\mathcal{T}$ *with respect to* $\mathcal{T}'$, and T is an *accumulation point* of $\mathcal{T}$ with respect to $\mathcal{T}'$.
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- **Theorem 14.** *For any family $\mathcal{T}$ of complete theories in at most countable language Σ, $\mathrm{Cl}_1(\mathcal{T}) = \mathrm{Cl}_E(\mathcal{T})$.*
- **Theorem 15.** *For any family $\mathcal{T}$ in at most countable language Σ the closure operator Cl_1 is transitive.*
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  (2) e_1-$Sp(\mathcal{T}) = 2^\omega$;
  (3) $\overline{RS}(\mathcal{T}) = \infty$.

[25] Markhabatov, N.D., Sudoplatov, S.V. Topologies, Ranks, and Closures for Families of Theories. II. Algebra and Logic 60:1, 57-80 (2021). https://doi.org/10.33048/alglog.2021.60.104

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- **Theorem 17.** *For any linearly $\subseteq$-ordered family $\mathcal{T}$, $\mathrm{Cl}_1(\mathcal{T})$ consists of unions for subfamilies of $\mathcal{T}$, and of intersections for countable subfamilies of $\mathcal{T}$ ordered by the type ω^*.*
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The family of permutation theories[27]

- Let a language Σ consist of the permutation f. Denote by $\mathcal{T}_\Sigma$ family of all permutation theories of language Σ.
- The following theorem shows that there is a family of permutations of theory, having a countable rank.
- **Theorem 19.** *For any countable ordinal α and natural $k \geq 1$ there exists a family $\mathcal{T} \subseteq \mathcal{T}_\Sigma$, such that $RS(\mathcal{T}) = \alpha$ and $ds(\mathcal{T}) = k$.*
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[27] Markhabatov, N. D. Ranks for families of permutation theories / N. D. Markhabatov // Известия Иркутского государственного университета. Серия: Математика. — 2019. — Т. 28. — С. 85—94.

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Locally free algebras[28]

- Let T be a complete first-order theory of locally free algebras of signature $\Sigma = \{f_1^{(n_1)}, \ldots, f_r^{(n_r)}, \ldots; c_1, \ldots, c_k, \ldots, \}$.
- **Theorem 21.** *An infinite locally free algebra A is pseudofinite if and only if the signature contains no more than one unary symbol and does not contain symbols arity $n > 1$.*
- **Corollary 22.** *A locally free groupoid is not pseudofinite.*

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Thank you!