Small models of fragments of Jonsson sets

A.R. Yeshkeyev, A.K. Issayeva, N.K. Shamatayeva

Karaganda Buketov University
Karagandy, Kazakhstan

ERLAGOL
14th International Summer School-Conference

JUNE 23-29, 2021

Outline

Notion of Jonsson theory.
Notion of semantic model and perfect Jonsson theory.
Notion of existentially prime theory.
Notion of (Γ_1, Γ_2)-cl atomic set and model.
Notion of almost (Γ_1, Γ_2)-cl atomic set and model.
Notion of almost-weakly (Γ_1, Γ_2)-cl atomic set and model.
Notion of (Γ_1, Γ_2)-cl- algebraically prime set and model.
Notion of almost (Γ_1, Γ_2)-cl algebraically prime set and model.
Notion of almost-weakly (Γ_1, Γ_2)-cl algebraically prime set and model.
Main result.

References

(1) Yeshkeyev, A.R., & Kassymetova, M.T(2016). *Yonsonovskie teorii i ikh klassy modelei*: [Jonsson theories and their classes of models]. Karaganda: Izdatelstvo KarHU [in Russian]

(2) Yeshkeyev A.R. ∇-cl - atomic and prime sets /A.R. Yeshkeyev, A.K. Issayeva// Bulletin of the Karaganda University. Mathematics series. - 2019. - Vol. 93. - No. 1. - P. 88-94. DOI 10.31489/2019M1/88-94

(3) Yeshkeyev A.R. The atomic definable subsets of semantic model /A.R. Yeshkeyev, A.K. Issayeva, N.M. Mussina // Bulletin of the Karaganda University. Mathematics series. 2019. - Vol. 94. - No. 2. - C.84-91. DOI 10.31489/2019M2/84-91

(4) Ешкеев А.Р. α - алгебраические простые модели и виды атомных моделей теории. /А.Р. Ешкеев, Т.Г. Мустафин // Теория алгебраических структур: Сб.начн.трудов.

Definition 1 [1].

A theory T is Jonsson if:

1. theory T has infinite models;
2. theory T is inductive;
3. theory T has the joint embedding property (JEP);
4. theory T has the property of amalgam (AP).

Definition 2 [1].

Model C of Jonsson theory T is said to be semantic model, if it is ω^+-homogeneous-universal.

Definition 3 [1].

The Jonsson theory T is said to be the perfect if its semantic model C is saturated.

Definition 4 [1].

The center of Jonsson theory T is said to be an elementary theory of the its semantic model. And denoted through T^*, i.e. $T^* = Th(C)$.

Definition 5 [1].

Let X be the set of Jonsson and M is the existentially closed model where $dcl(X) = T_M$, we considered $Th_{\forall\exists}(M) = T_M$. T_M is called the Jonsson fragment of Jonsson's set.

Definition 6 [1].

Model A of the theory T is called existentially closed if for any model B and any existential formula $\varphi(\overline{x})$ with constants of A we have $A \models \exists\overline{x}\varphi(\overline{x})$ provided that A is a submodel of B and $B \models \exists\overline{x}\varphi(\overline{x})$.

Definition 7 [1].

The inductive theory T is called the existentially prime if:

1. it has an algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T;
2. class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq \varnothing$.

Definition 8 [2].

A set A will be called (Γ_1, Γ_2)-cl atomic in T, if
1)$\forall \bar{a} \in A$, $\exists \varphi(\bar{x}) \in \Gamma_1$ such that $\mathfrak{A} \models \varphi(\bar{a})$;
2) $\varphi(\bar{x})$ generates $t^{\mathfrak{A}}_{\Gamma \cup \Gamma^*}(\bar{a})$;
3) $cl(A) = M, M \in E_T$, where E_T class of the existentially closed models of the theory T;
and obtained model M is said to be (Γ_1, Γ_2)-cl atomic model of the theory T.

Definition 9.

A set A is said to be almost (Γ_1, Γ_2)-cl atomic in the theory T, if $\forall \bar{a} \in A \; \exists \varphi(\bar{x}) \in \Gamma_1$ such that:
1) $\varphi(\bar{x}) \cup T$ is consistent;
2) $\varphi(\bar{x})$ generates $t^{\mathfrak{A}}_{\Gamma_2 \cup \Gamma_2^*}(\bar{a})$;
3) $cl(A) = M, M \in E_T$, where E_T class of existentially closed models of the theory T;
and obtained model M is said to be almost (Γ_1, Γ_2)-cl atomic model of the theory T.

Definition 10.

A set A is said to be the almost-weakly (Γ_1, Γ_2)-cl atomic in T, if for any $\bar{a} \in A$ there exists a formula $\varphi(\bar{x}) \in \Gamma_1$ such that:
1) $\varphi(\bar{x}) \cup T$ is consistent;
2) $\varphi(\bar{x})$ generates $t^{\mathfrak{A}}_{\Gamma_2}(\bar{a})$;
3) $cl(A) = M, M \in E_T$, where E_T class of existentially closed models of theory T;
and obtained model M is said to be almost-weakly (Γ_1, Γ_2) - cl atomic model of the theory T.

Definition 11 [3].

A set A is said to be (Γ_1, Γ_2) -*cl*- algebraically prime of the theory T, if $cl(A) = M$, M is (Γ_1, Γ_2)-*cl* atomic model of the theory T, $M \in E_T \cap AP_T$, where $AP_T \bigcap E_T \neq \varnothing$
and obtained model M is said to be (Γ_1, Γ_2)-*cl* algebraically prime of the theory T.

Definition 12.

A set A is said to be the almost (Γ_1, Γ_2)-*cl* algebraically prime of the theory T, if $cl(A) = M$, M is been almost (Γ_1, Γ_2)-*cl* atomic model of the theory T, $M \in E_T \cap AP_T$, where $AP_T \bigcap E_T \neq \varnothing$
and obtained model M is said to be almost (Γ_1, Γ_2)-*cl* algebraically prime of the theory T.

Definition 13.

A set A is said to be almost-weakly (Γ_1, Γ_2)-cl algebraically prime of theory T, if $cl(A) = M$, M is been almost-weakly (Γ_1, Γ_2)-cl atomic model of the theory T, $M \in E_T \cap AP_T$, где $AP_T \bigcap E_T \neq \varnothing$ and obtained model M is said to be almost-weakly (Γ_1, Γ_2)-cl algebraically prime of the theory T.

For the convenience of expression
"$\mathfrak{A}$ is (Γ_1, Γ_2)-*cl*-atomic model of the theory T ";
"$\mathfrak{A}$ is weakly (Γ_1, Γ_2)-*cl*-atomic model of theory T ";
"$\mathfrak{A}$ is an almost (Γ_1, Γ_2)-*cl*-atomic model of theory T ";
"$\mathfrak{A}$ is an almost-weakly (Γ_1, Γ_2)-*cl*-atomic model of theory T";
and denote by (1),(2),(3),(4), respectively.

Definition 14.

Let $\Phi(x_1...x_\kappa)$ be the some set of formulas of the language L from variables $x_1...x_\kappa$. We say that Γ_1 locally omitted Φ, if for any formula consistent with T formulas $\varphi(x_1...x_\kappa) \in \Gamma_1$ there is such a formula $\theta(x_1...x_\kappa) \in \Phi$ such that $\varphi \wedge \neg\theta$ consistent with T.

Definition 15.

Let t_1 be the Γ_1-type, t_2 be the Γ_2-type, then they say that t_1 and t_2 T-equivalent if $T \cup t_1 \vdash t_2$ & $T \cup t_2 \vdash t_1$. In this case, write $t_1 \sim_T t_2$.

Lemma 1[4].

Let T be the perfect Jonsson theory complete for Π_2 sentences and $\mathfrak{A} \models T$, then there is a model $\mathfrak{B}$, such that:
1) $\mathfrak{B} \models T$;
2) $\mathfrak{A}$ is the isomorphically embeddable in $\mathfrak{B}$;
3) for any $\bar{b} \in B$ $t^{\mathfrak{B}}_{\Sigma_1}(\bar{b}) \sim_T t^{\mathfrak{B}}_{\Sigma_2}(\bar{b})$

Definition 16 [4].

1) α-type is called any set of formulas consistent with the theory T, the free variables of which are found in $\bar{x}$, where $L(x_1...x_n) = \alpha$;
2) ω-type ρ is called Γ-ω-type, if $\rho \subseteq \Gamma$;
3) Γ-ω-type ρ is called Γ_1-principle type, if there exists such a sequence $\langle \psi_n(\bar{x}^n) : 1 \leq n < \omega \rangle$ Γ_1-formulas, such that:
a) $T \cup \psi_n(\bar{x}^n)$ is consistent, $1 \leq n < \omega$;
b) $\psi_n(\bar{x}^n)$ generates $\rho \upharpoonright \bar{x}^n$, where $\rho \upharpoonright \bar{x}^n$ is set of formulas from ρ, the free variables of which are among $(x_1, ..., x_n)$, $1 \leq n < \omega$;
c) $T \vdash \psi_n(\bar{x}^n) \leftrightarrow \exists \bar{x}_{n+1} \psi_{n+1}(\bar{x}^{n+1})$, $1 \leq n < \omega$.

Definition 17.

A model $\mathfrak{A}$ of the theory T is said to be the **fine almost-weakly** (Γ_1, Γ_2)-*cl* atomic model of T, if each tuple of ω elements $\mathfrak{A}$ implements Γ_1-principle type Γ_2 ω-type.

Theorem 1.

Let T be the perfect Jonsson theory complete for Π_1- sentences and has fine almost-weakly (Σ, Σ)-cl-atomic model. Then the following conditions are equivalent:
1) $\mathfrak{A}$ is the (Σ, Σ)-cl-algebraically prime model of the theory T.
2) $\mathfrak{A}$ is the fine almost-weakly (Σ, Σ)-cl-atomic model of the theory T.

Thanks a lot for your attention!